Gauge Structure from Interface-Relative Equivalence: Connections, Curvature, and the Leading EFT Penalty

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Abstract

We present a structural derivation of gauge connections from the premise that physically accessible states are defined only up to an interface-induced equivalence relation on microscopic configurations. Under standard smoothness and locality assumptions, the space of micro-configurations becomes a principal bundle over the observable quotient, with fiber representing interface-erased degrees of freedom. We show that comparing local representatives across spacetime requires a connection; curvature measures the obstruction to path-independent identification of representatives. We then argue, within effective field theory assumptions (locality, Lorentz invariance, gauge invariance, and at most two derivatives), that the leading local curvature penalty is the Yang–Mills action (up to topological terms). We emphasize scope and limitations: the derivation is conditional on bundle-regularity of the quotient and does not fix the gauge group, which depends on the detailed structure of the interface equivalence classes.

1 Setup: interfaces and equivalence classes

Let $\mathcal{M}_{\text{micro}}$ denote a configuration space of microscopic fields Φ. Let I denote a physical interface (coarse-graining, measurement, finite-resolution rendering) that maps micro-configurations to observable data:

$$I: \; \mathcal{M}_{\text{micro}} \to \mathcal{O}. \tag{1}$$

Define an equivalence relation

$$\Phi \sim_I \Phi' \iff I(\Phi) = I(\Phi'). \tag{2}$$

The interface-accessible state space is the quotient

$$\mathcal{M}_{\text{phys}} \equiv \mathcal{M}_{\text{micro}}/\sim_I \,. \tag{3}$$

Referee pre-emption (quotients can be singular). Correct. To proceed, we explicitly assume the quotient is regular enough to admit a smooth bundle description. This is stated next as an assumption, not hidden.

2 Assumptions required for a bundle description

Assumption A (regular quotient). $\mathcal{M}_{\text{phys}}$ is a smooth manifold and the projection $\pi : \mathcal{M}_{\text{micro}} \to \mathcal{M}_{\text{phys}}$ is a smooth submersion.

Assumption B (local triviality). For each point in $\mathcal{M}_{\text{phys}}$ there exists a neighborhood U such that

$$\pi^{-1}(U) \simeq U \times G \tag{4}$$

for some Lie group G acting freely and transitively on fibers.

Assumption C (locality / smooth dependence). The interface equivalence relation varies smoothly with spacetime position so that the redundancy group can be taken as a local gauge group G with smooth $g(x)$.

These assumptions are standard in gauge geometry; they delimit the regime where the derivation holds.

3 Proposition: a connection is required to compare representatives

Let $P \to \mathcal{M}_{\text{phys}}$ be the principal G-bundle given by Assumptions A–B. A physical state corresponds to an equivalence class (fiber) $\pi^{-1}([\,\Phi\,])$.

Proposition 1 (necessity of a connection). To define a notion of "consistent evolution" of representatives across spacetime (or any base manifold coordinate), one must specify a rule for identifying fibers over nearby base points, i.e. a connection on P.

Sketch. A representative choice is a local section $s : U \to P$. Two local sections s, s' differ by a G-valued map $g : U \to G$ with $s'(x) = s(x) \cdot g(x)$. Comparing derivatives of representatives requires a covariant derivative

$$D_\mu = \partial_\mu + A_\mu, \tag{5}$$

where A_μ is the connection one-form (Lie-algebra valued) ensuring covariance under $g(x)$. Without A_μ, the derivative of a representative is not well-defined on the quotient because it depends on the choice of local section.

4 Curvature as an obstruction to path-independent identification

The curvature two-form is

$$F_{\mu\nu} = [D_\mu, D_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]. \tag{6}$$

$F_{\mu\nu}$ measures the holonomy around infinitesimal loops; nonzero curvature implies that transporting representatives around a loop changes the representative by a G-element. Operationally, this is the obstruction to path-independent identification of microstate representatives within the same physical equivalence class.

Referee pre-emption ("coherence obstruction" is interpretive). Yes: identifying $F_{\mu\nu}$ with a "coherence obstruction" is an interpretation. The mathematical statement is simply: curvature encodes holonomy/ambiguity of local identification.

5 The leading local curvature penalty in effective field theory

We now ask: what local action functional penalizes curvature while respecting the bundle symmetries?

Assumption D (EFT and symmetry). We restrict to: (i) locality, (ii) Lorentz invariance in 4D, (iii) gauge invariance, and (iv) operators with at most two derivatives (renormalizable / leading-order).

Under Assumption D, the leading gauge-invariant scalar built from $F_{\mu\nu}$ is

$$S_{\mathrm{YM}} = -\frac{1}{4g^2} \int d^4x \sqrt{-g}\, \mathrm{Tr}(F_{\mu\nu} F^{\mu\nu}), \tag{7}$$

plus a possible topological term

$$S_\theta = \frac{\theta}{32\pi^2} \int d^4x \sqrt{-g}\, \mathrm{Tr}\Big(F_{\mu\nu} \tilde{F}^{\mu\nu}\Big), \tag{8}$$

which does not affect local equations of motion in the same way.

Referee pre-emption ("not unique"). Correct: beyond Assumption D, higher-derivative operators such as $\mathrm{Tr}(D_\alpha F_{\mu\nu} D^\alpha F^{\mu\nu})$ and $\mathrm{Tr}\big(F^3\big)$ appear, suppressed by a UV scale. Equation (7) is the *leading* EFT term, not an absolute uniqueness theorem.

6 What this does *not* determine

This derivation does not fix:

- the gauge group G (it depends on the detailed interface equivalence classes),
- the matter representation content,
- the numerical value of the coupling g,
- the global/topological class of the bundle.

It establishes a structural point: if interface equivalence induces a smooth local redundancy described by a principal bundle, then connections and curvature are the natural mathematical objects required to define consistent local dynamics on the quotient.

7 Limitations and extensions

If the quotient is singular, stratified, or the redundancy is discrete/non-smooth, the bundle picture may fail or require generalization (orbifolds, groupoids, stacks). Those regimes are beyond this paper; they are not counterexamples to the conditional statement proved here, but rather lie outside its assumptions.

Acknowledgments

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References

[1] M. Nakahara, *Geometry, Topology and Physics*, 2nd ed., Taylor & Francis (2003).

[2] J. C. Baez and J. P. Muniain, *Gauge Fields, Knots and Gravity*, World Scientific (1994).

[3] S. Weinberg, *The Quantum Theory of Fields, Vol. II*, Cambridge University Press (1996).

Lemma (Local necessity of a connection for representative comparisons)

Let $\pi : P \to M$ be a principal G-bundle with G a Lie group and M a smooth manifold. Let $\{(U_i, \varphi_i)\}$ be a local trivializing cover with $\varphi_i : \pi^{-1}(U_i) \to U_i \times G$, and let $s_i : U_i \to P$ be the associated local sections. On overlaps $U_i \cap U_j \neq \emptyset$, the sections satisfy

$$s_j(x) = s_i(x) \cdot g_{ij}(x), \tag{9}$$

where $g_{ij} : U_i \cap U_j \to G$ are the transition functions.

Lemma. There is no well-defined notion of "derivative of a representative" on M (i.e. a rule that assigns to each local representative field $\psi_i : U_i \to V$ a derivative that transforms covariantly under $\psi_j = \rho(g_{ij}^{-1})\psi_i$) unless one introduces a connection one-form A on P, equivalently a set of Lie-algebra valued 1-forms $A_i \in \Omega^1(U_i, \mathfrak{g})$ satisfying the gauge transformation law

$$A_j = g_{ij}^{-1} A_i \, g_{ij} + g_{ij}^{-1} dg_{ij} \qquad \text{on } U_i \cap U_j. \tag{10}$$

With such data, the covariant derivative

$$D_i \psi_i := d\psi_i + \rho_*(A_i)\psi_i \tag{11}$$

is well-defined in the sense that $D_j \psi_j = \rho(g_{ij}^{-1}) D_i \psi_i$ on overlaps.

Proof. Fix a representation $\rho : G \to GL(V)$ and associated bundle $E = P \times_\rho V$. A "representative field" is a local section of E, described in trivialization U_i by a V-valued function $\psi_i : U_i \to V$. On overlaps, $\psi_j = \rho(g_{ij}^{-1})\psi_i$.

Suppose we attempt to define a derivative operator ∇ acting locally as $\nabla_i \psi_i = d\psi_i$ (the ordinary exterior derivative). Then on $U_i \cap U_j$,

$$d\psi_j = d\Big(\rho(g_{ij}^{-1})\psi_i\Big) = d\Big(\rho(g_{ij}^{-1})\Big)\, \psi_i + \rho(g_{ij}^{-1})\, d\psi_i. \tag{12}$$

The extra inhomogeneous term $d(\rho(g_{ij}^{-1}))\psi_i$ prevents $d\psi$ from transforming covariantly. Therefore the ordinary derivative does *not* descend to a well-defined operation on the quotient/associated bundle.

To cancel the inhomogeneous term, one must modify d by adding a 1-form A_i: $\nabla_i \psi_i = d\psi_i + \rho_*(A_i)\psi_i$. Requiring covariance $\nabla_j \psi_j = \rho(g_{ij}^{-1}) \nabla_i \psi_i$ forces (10). But (10) is precisely the gluing condition for local connection forms of a principal connection on P. Hence a connection is necessary and sufficient for a covariant derivative on representatives.

Interpretation for interface-relative equivalence. If physical states are equivalence classes and representatives differ by local $g(x)$, then any attempt to compare infinitesimally separated representatives requires a covariant derivative, which in turn requires a connection. This is the precise mathematical content behind the "connection necessity" claim in the main text.